Algebra I CPA Summer Packet Notes

Please use this packet if you have questions with content, or don't remember how to complete problems. This packet contains helpful websites to visit and additional notes for each topic in your summer packet.

Helpful Resources and Websites:

1. http://www.math.com/homeworkhelp/PreAlgebra.html - for help with adding/subtracting/multiplying/dividing integers, fractions, and decimals
2. http://www.math.com/homeworkhelp/Algebra.html - for help with order of operations, combining like terms, solving equations
3. http://www.mathgoodies.com/lessons/ - for help with percentages, decimals, fractions, probability, statistics, and PEMDAS
4. http://www.teacherschoice.com.au/mathematics_how-to_library.htm - for help with evaluating and solving equations
5. http://www.algebrahelp.com/ - a great resource for help in all topics in algebra as well as basic skills
6. http://www.freemathhelp.com/algebra-help.html - includes text and video lessons includes text and video lessons from an array of algebra topics including equations, inequalities, and polynomials.
7. http://mathforum.org/library/drmath/drmath.middle.html - An achieve of questions and answers by Dr. Math
8. http://www.purplemath.com/modules/index.htm - excellent resource for topics from pre-algebra to advanced algebra
9. www.brainpop.com – has a free trial version which includes video tutorials and more
10. www.khanacademy.com - free video tutorials on various topics

Notes for Fractions

A fraction is in **simplest form** if its numerator and denominator have a greatest common factor of 1. To simplify a fraction, divide the numerator and denominator by their greatest common factor.

Example: $\frac{28}{63} = \frac{28\div7}{63\div7} = \frac{4}{9}$

Two rewrite a **mixed fraction to improper**, multiply the denominator and the whole number, then add the numerator. The denominator stays the same.

Example: $2\frac{4}{5} =$; $5 \cdot 2 + 4 = 14$; $= \frac{14}{5}$

Two numbers are **reciprocals** of each other if their product is 1. Every number except 0 has a reciprocal. To find the reciprocal of a number, write the number as a fraction. Then interchange the numerator and the denominator.

Example: $\frac{2}{3} \times \frac{3}{2} = 1$; so $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

To **add or subtract two fractions** with the same denominator, add or subtract the numerators. To add or subtract two fractions with different denominators, find the least common denominator to create equivalent fractions and then add or subtract the numerators.

Example 1: $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$

Example 2: $\frac{3}{5} - \frac{1}{2} =$; *Let 10 be the least common denominator*

$\frac{3}{5} = \frac{3\cdot2}{5\cdot2} = \frac{6}{10}$

$\frac{1}{2} = \frac{1\cdot5}{2\cdot5} = \frac{5}{10}$

$\frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

To **multiply** two fractions with same and different denominators, multiply the denominators together and multiply numerators together. Reduce/simplify the answer if necessary

Example: $\frac{3}{5} \times \frac{4}{3} = \frac{12}{15} = \frac{4}{5}$

To **divide** two fractions with same and different denominators, change the division sign to multiplication and find the reciprocal of the second fraction. Then follow the directions for multiplying fractions.

Example: $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$

Notes for Comparing and Ordering Numbers

When you compare two numbers, *a* and *b*, a is either **less than**, **equal to**, or **greater than** b. To compare two whole numbers or decimals, compare the digits of the two numbers from left to right. Find the first place in which the digits are different. To compare two fractions, find the least common denominator and compare the numerators.

a is less than b.	$a < b$
a is equal to b.	$a = b$
a is greater than b.	$a > b$

EXAMPLES

Compare the two numbers. Write the answer using <, >, or =.

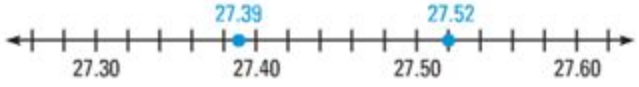
a. 27.52 and 27.39 **b.** −4.5 and −4.25

SOLUTION

a. 27.52 and 27.39

ANSWER ▶ 5 > 3, so 27.52 > 27.39.

You can picture this on a number line. The numbers on a number line increase from left to right.



27.52 is *greater* than 27.39.

27.52 is to the *right* of 27.39.

b. Begin by graphing −4.5 and −4.25 on a number line.

−4.5 is *less* than −4.25.

−4.5 is to the *left* of −4.25.

ANSWER ▶ From the number line, −4.5 is to the left of −4.25, so −4.5 < −4.25.

EXAMPLE **Write the numbers $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{12}$ in order from least to greatest.**

SOLUTION

The LCD of the fractions is 24.

$$\frac{3}{4} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24} \qquad \frac{7}{8} = \frac{7 \cdot 3}{8 \cdot 3} = \frac{21}{24} \qquad \frac{5}{12} = \frac{5 \cdot 2}{12 \cdot 2} = \frac{10}{24}$$

Compare the numerators: $10 < 18 < 21$, so $\frac{10}{24} < \frac{18}{24} < \frac{21}{24}$, or $\frac{5}{12} < \frac{3}{4} < \frac{7}{8}$.

ANSWER ▶ In order from least to greatest, the fractions are $\frac{5}{12}$, $\frac{3}{4}$, and $\frac{7}{8}$.

Notes for Evaluating Expressions

To evaluate a numerical expressions, follow the order of operation rules:

P - Parenthesis
E - Exponent
M/D - Multiplication/Division
A/S - Addition/Subtraction

EXAMPLE **Evaluate the expression $2 - (4 - 7)^2 \div (-6)$.**

Solution

$2 - (4 - 7)^2 \div (-6) = 2 - (-3)^2 \div (-6)$	Evaluate within parentheses.
$= 2 - 9 \div (-6)$	Evaluate the power: $(-3)^2 = 9$.
$= 2 - (-1.5)$	Do the division $9 \div (-6)$.
$= 3.5$	Do the subtraction $2 - (-1.5)$.

To evaluate variable expressions, substitute values into the expression and follow the order of operation rules.

EXAMPLE **Evaluate the expression $x^2 + x - 18$ when $x = 5$.**

Solution

$x^2 + x - 18 = 5^2 + 5 - 18$	Substitute 5 for each x.
$= 25 + 5 - 18$	Evaluate the power: $5^2 = 25$.
$= 12$	Add and subtract from left to right.

Notes for Distributive Property

Here are the four forms of the distributive property.

$a(b + c) = ab + ac$ $\qquad$ $(b + c)a = ba + ca$

$a(b - c) = ab - ac$ $\qquad$ $(b - c)a = ba - ca$

EXAMPLES **Use the distributive property to write the expression without parentheses.**

a. $x(x + 4)$ $\qquad$ **b.** $5 - (n - 2)$

Solution

a. $x(x + 4) = x(x) + x(4) = x^2 + 4x$

b. $5 - (n - 2) = 5 + (-1)(n - 2) = 5 + (-1)(n) + (-1)(-2) = 5 - n + 2 = 7 - n$

Terms are parents of an expression separated by addition and subtraction signs. **Like terms** have the same variable raised to the same exponent power. Numbers with no variables are considered like terms. You can use the distributive property to combine like terms.

EXAMPLES **Simplify the expression.**

a. $-4x + 7x$ $\qquad$ **b.** $2(x + y) - (4 - y)x$

Solution

a. $-4x + 7x = (-4 + 7)x = 3x$

b. $2(x + y) - (4 - y)x = 2x + 2y - 4x + xy$ Write without parentheses.

$= (2 - 4)x + 2y + xy$ Group and combine like terms.

$= -2x + 2y + xy$ Simplify within parentheses.

Notes for Solving Multi-Step Equations

Sometimes solving an equation requires more than one step. Simplify one or both sides of the equation first by combining like terms or using distributive property.

EXAMPLES **Solve the equation.**

a. $y - 7 = 3$ **b.** $x + 6 = -2$ **c.** $\frac{n}{5} = 30$ **d.** $12 = -4c$

Solution Choose an operation to perform that will leave the variable alone on one side. Check your solution by substituting it back into the original equation.

a. Add 7 to each side.

$$y - 7 = 3$$
$$y - 7 + 7 = 3 + 7$$
$$y = 10$$

CHECK ✓ $10 - 7 = 3$

b. Subtract 6 from each side.

$$x + 6 = -2$$
$$x + 6 - 6 = -2 - 6$$
$$x = -8$$

CHECK ✓ $-8 + 6 = -2$

c. Multiply each side by 5.

$$\frac{n}{5} = 30$$
$$5 \cdot \frac{n}{5} = 5 \cdot 30$$
$$n = 150$$

CHECK ✓ $\frac{150}{5} = 30$

d. Divide each side by -4.

$$12 = -4c$$
$$\frac{12}{-4} = \frac{-4c}{-4}$$
$$-3 = c$$

CHECK ✓ $12 = -4(-3)$

To solve equations with variables on both sides, you can use the properties of equality and inverse operations to write a series of simpler equivalent equations.

EXAMPLES **Solve the equation.**

Example 1:

Solve $2(m-2) + 5m = 13 - 2(3m + 2)$

Solution:

$2(m-2) + 5m = 13 - 2(3m + 2)$

$2m - 4 + 5m = 13 - 6m - 4$	Distribute.
$7m - 4 = -6m + 9$	Add the terms with variables together on the left side and the constants on the right side to combine like terms.
$7m - 4 + 6m = -6m + 9 + 6m$	To move the variables to the left side, add $6m$ to each side.
$13m - 4 = 9$	Simplify.
$13m - 4 + 4 = 9 + 4$	To get the variable term alone on the left, add 4 to each side.
$13m = 13$	Simplify.
$\frac{13m}{13} = \frac{13}{13}$	Divide each side by 13 since x is being multiplied by 13 on the left side. This isolates x.
$m = 1$	Simplify.

Notes for Identifying Slope and Y-Intercept from Linear Equations

The equation of a line written in the form **y = mx + b** is in **slope-intercept form.** The slope of the line is **m** and the y-intercept is the point **(0, b)**.

Example 1: $y = 2x + 6$

Since m = 2 and b = 6; that means the slope is 2 and y-intercept is 6

Example 2: $y = -\frac{1}{2}x - 5$

Since $m = -\frac{1}{2}$ and b = -5; that means the slope is $-\frac{1}{2}$ and y-intercept is -5

Notes for Graphing Lines in Slope-Intercept Form

Remember the equation of a line written in the form **y = mx + b** is in **slope-intercept form.** The slope of the line is **m** and the y-intercept is the point **(0, b)**.

Example 1:

Graph $y = 2x + 1$. Identify the slope and the y-intercept.

Step 1: Find the slope and y-intercept.

m = 2 y-intercept: (0, 1)

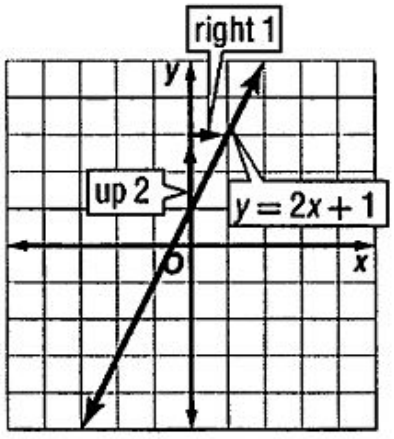
Step 2: Graph the y-intercept (0, 1)

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$m = \frac{2}{1}$ ← change in y : up 2 units
← change in x : right 1 unit

Remember: slope is rise over run!

Step 4: Draw the line.



Notes for Using Proportion Method to Solve Percent Problems

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

PERCENT -- the number with the percent sign (%).
PART -- the number with the word ***is***.
WHOLE -- the number with the word ***of***.

Example:

What number is 75% of 4? (*or* Find 75% of 4.)

The PERCENT *always* goes over 100.
(It's a part of the whole 100%.)
4 appears with the word ***of***:
It's the WHOLE and goes on the bottom.

$$\frac{\text{part}}{4} = \frac{75}{100}$$

We're trying to find the missing PART (on the top).

In a proportion the cross-products are equal: So 4 times 75 is equal to 100 times the PART.

The missing PART equals 4 times 75 divided by 100.

(Multiply the two opposite corners with numbers; then divide by the other number.)

4 times 75 = 100 times *the part*
300 = 100 times *the part*
300/100 = 100/100 times *the part*
3 = *the part*

$$\frac{\quad}{4} = \frac{75}{100}$$